Heat Transfer to Variable Property Fluids in Turbulent Pipe Flow

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The paper is concerned with heat transfer to (or from) fluids in turbulent flow through smooth pipes with moderate property variations. The objectives were to extend constant property eddy diffusivity models to variable property pipe flow in a systematic way, and to compute the effects of the variations of c_p, k, ρ, and μ separately and in combination for ranges of *Re* and *Pr*. This has been achieved to an extent but with the growing realization that eddy diffusivity models, even for constant properties, lack a logical foundation so that their extension to variable properties must be equally arbitrary. Nevertheless theoretical results derived from the model proposed in the paper should yield design predictions at least as reliable as could be obtained by direct use of the multiplicity of unsatisfactory and contradictory experimental correlations.

NOTATION

a, A^+, b, B^+, c, C	constants
A	cross-sectional area for flow
c_p	specific heat at constant pressure
G	mass velocity $= \dot{m}/A$
k	thermal conductivity
m, n	exponents
$\dot{m}$	mass flow rate
r	radius
r_0^+	dimensionless pipe radius $= r_0 u^*/\mu_w$
t	temperature
t_r	reference temperature
T	absolute temperature
t^+	dimensionless temperature $= \dfrac{(t_w - t_b)c_{pw}\tau_w}{q_w u^*}$
u	velocity in axial direction
u^*	friction velocity $\sqrt{(\tau_w/\rho_w)}$
u^+	dimensionless velocity $= u/u^*$
y	co-ordinate normal to wall
y^+	dimensionless distance from wall $\dfrac{yu^*}{\mu_w}$
y_1^+, y_2^+	edges of wall and logarithmic layers respectively
α, β, γ	exponents
ε_m	eddy diffusivity of momentum
ε_h	eddy diffusivity of heat
μ	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density
τ	shear stress
Δc_p	specific heat difference $= c_{pw} - c_{pb}$
Δk	thermal conductivity difference $= k_w - k_b$

Non-dimensional parameters

f	Fanning friction factor $2\tau_w\rho_b/G^2$
f_p	Friction factor given by Prandtl law for smooth pipes
Nu	Nusselt number hD/k
ΔNu_b	$Nu_b - Nu_{b,0}$
Pr	Prandtl number $c_p\mu/k$
Pr_t	Turbulent Prandtl number $\varepsilon_m/\varepsilon_h$
Re	Reynolds number GD/μ

Subscripts

0	constant property value
b	bulk
L	local physical properties applied wherever they occur
r	at reference temperature t_r
t	turbulent
w	wall

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INTRODUCTION

The scope of this paper is heat transfer to (or from) fluids in turbulent flow through smooth pipes with wall–fluid temperature differences large enough to produce significant variations of the relevant physical properties c_p, k, ρ, and μ. A great deal of work, theoretical as well as experimental, has already been reported but even for moderate property variations, to which this paper is restricted, where buoyancy, acceleration, and compressibility effects can be ignored, no general correlations exist which could be used with confidence for predictions of friction and heat transfer.

A knowledge of the effects of the variations of c_p, k, ρ, and μ, separately and in combination, would be a useful contribution to the development of a reliable correlation. For liquids far from their critical points the temperature dependence of viscosity is very much greater than the dependence of c_p, k, and ρ. This permits the effect of the viscosity variation on *Nu* and f to be studied experimentally. There are no fluids which lend themselves to the study of the variation of c_p or k or ρ in isolation. Moreover, for gases, where c_p, k, ρ, and μ may all vary significantly, the variations from one gas to another are not sufficiently different to enable the effects

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of the different properties to be isolated by a series of experiments with different gases.

Although many correlations have been proposed which purport to represent the dependence of Nu and f on Re, Pr and the variations of some or all of the four physical properties, all of the correlations contain highly arbitrary features and it is not possible to regard them as more than summaries of particular sets of experimental data. Substantial discrepancies are found when predictions are made from alternative correlations. Some of the correlations contain features which contradict reasonable expectations (**1**).

A number of theoretical, more strictly empirical and computational, studies have also been reported but these have tended to be limited and unsystematic, i.e., restricted to small ranges of Re and Pr and often to particular fluids. The approach has been to extend to variable properties the eddy diffusivity models developed for constant property pipe flow but generally without systematic investigation of alternative ways of making the extension.

The objectives of the present investigation were to investigate in a more systematic way the alternative methods of extending constant property eddy diffusivity models to variable property pipe flow and to assess the uncertainties involved; to compute the effect of the variation of c_p, k, ρ, and μ separately, including the form of the variation with temperature, and the combined effect of two or more property variations; and to extend the calculations to adequate ranges of Re and Pr.

EDDY DIFFUSIVITY MODELS FOR CONSTANT PROPERTY PIPE FLOW

The eddy diffusivity models for variable properties start from the constant property case and the first step is therefore to establish a suitable constant property model for ranges of Re and Pr.

A constant property formulation for ε_m/ν should satisfy the following criteria arising from theoretical findings and experimental observations:

(1) ε_m/ν should vary continuously over the pipe radius to yield a smooth velocity distribution, and should be non-zero up to the pipe centre line to ensure that the velocity distribution has zero gradient.
(2) As $y^+ \to 0$, ε_m/ν should vary as y^{+3} (**2, 3, 4, 5, 6**). This becomes significant for very high Prandtl numbers.
(3) As a result of his temperature measurements, Sleicher (**7**) suggested $\varepsilon_m/\nu \propto y^{+2}$ in the range $10 < y^+ < 45$.
(4) The predictions of the velocity profile and of the friction factor should agree with experimental data over a wide range of Re.

Finally

(5) the combination of ε_m/ν and a formulation for Pr_t should yield temperature profiles and values of Nu in agreement with experimental data, although a recent review (**8**) includes the observation that 'the variability of the evidence currently available prevents us from using it as anything more than a very coarse filter'.

The expressions adopted for ε_m/ν were:

$$\frac{\varepsilon_m}{\nu} = \frac{ay^{+3}}{(1 + by^+ + cy^{+2})^{1/2}} \qquad 0 < y^+ < y_1^+ \quad (1)$$

$$= y^+(1 - y^+/r_0^+) \cdot \frac{1}{2{\cdot}46} - 1 \qquad y_1^+ < y^+ < y_2^+ \quad (2)$$

$$= y_2^+(1 - y_2^+/r_0^+) \cdot \frac{1}{2{\cdot}46} - 1 \qquad y_2^+ < y^+ < r_0^+ \quad (3)$$

For Pr_t the simple expression of Na and Habib (**9**) was used (but with reoptimization of the constants):

$$Pr_t = \frac{1 - \exp(-y^+/A^+)}{1 - \exp(-y^+/B^+)} \qquad (4)$$

Reoptimization yielded the following values for the adjustable constants in eqs. (1) to (4):

$$a = 4{\cdot}8 \times 10^{-4}$$
$$b = -0{\cdot}04$$
$$c = 1{\cdot}2 \times 10^{-3}$$
$$A^+ = 35$$
$$B^+ = 34{\cdot}2 \times Pr^{-0{\cdot}025}$$

y_1^+ is the intercept of eqs. (1) and (2)

$$y_2^+/r_0^+ = 0{\cdot}25 - 1{\cdot}3 \times 10^{-2} \log_{10} Re$$

The preceding expressions, and calculations of velocity distributions, friction factors, etc., deriving from them, have been compared with experimental data and other theoretical results. A selection from the comparisons made is shown in Fig. 1 for ε_m/ν, Figs. 2a and 2b for u^+ against y^+ for the wall region and core respectively, and Fig. 3 where the percentage deviation of

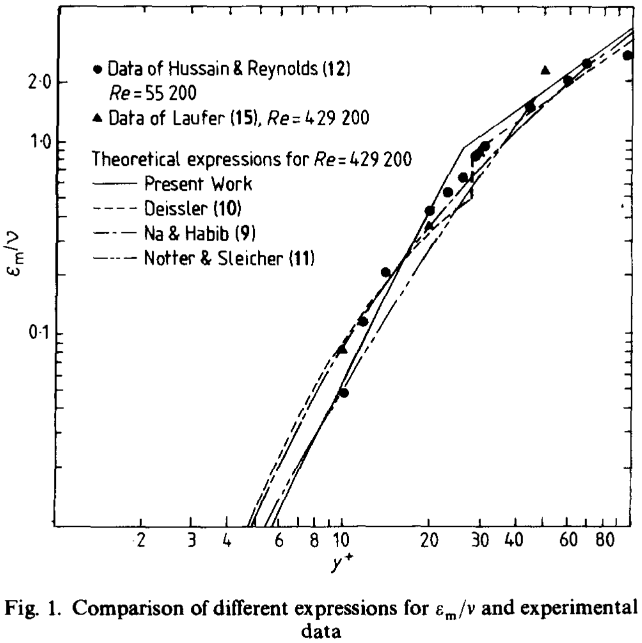

Fig. 1. Comparison of different expressions for ε_m/ν and experimental data

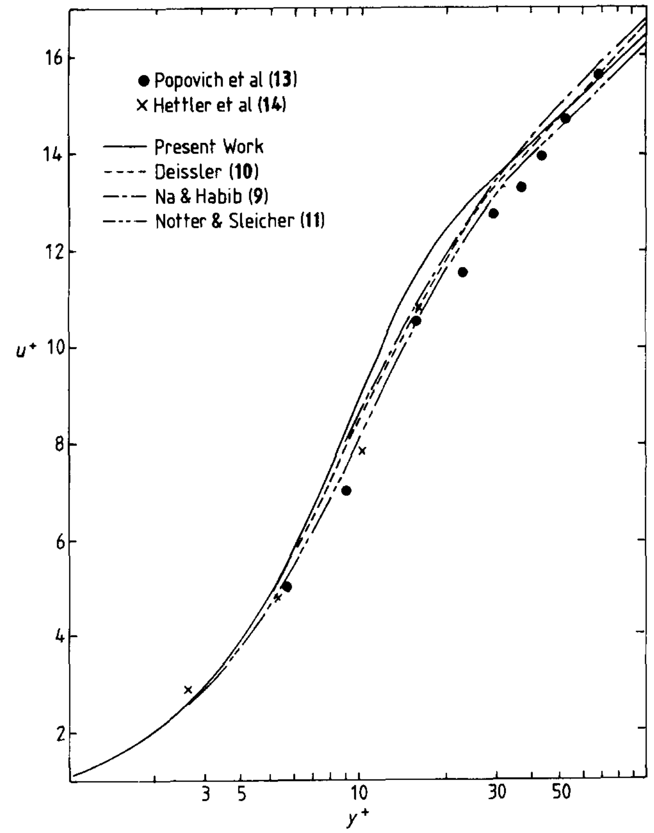

Fig. 2a. Experimental and theoretical velocity distributions in the wall region, $Re = 13\,100$

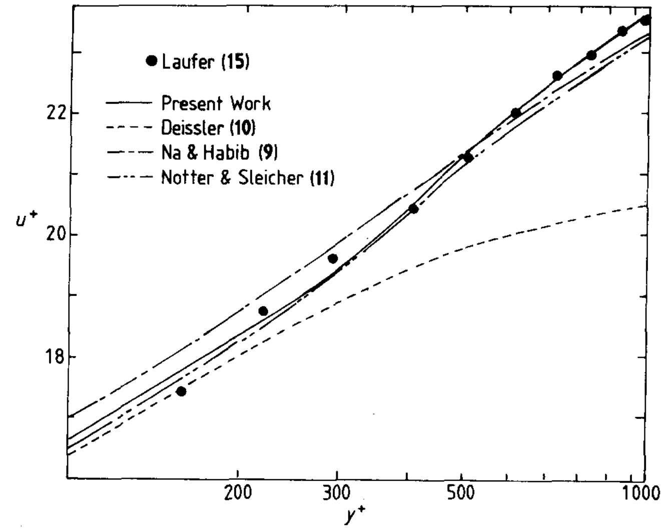

Fig. 2b. Experimental and theoretical velocity distributions in the core region $Re = 40\,260$

the friction factor from the well-known logarithmic or Prandtl formula

$$\frac{1}{\sqrt{f_p}} = 4 \log_{10} (Re\sqrt{f_p}) - 0{\cdot}4 \qquad (5)$$

is plotted for Re from 10^4 to 10^6. The experimental friction factors of Allen and Eckert (**16**) have been highlighted because of the precision of that investigation; particular weight is placed on their heat transfer results in the later discussion of the effect of viscosity variations. Considerable weight is attached to the friction factor comparison because of the precision with which f can be determined experimentally and it is observed that the present predictions are within 3 per cent of the Prandtl law from $Re = 2 \times 10^4$ to beyond 10^6.

Dimensionless temperature distributions are compared in Fig. 4 for three different Prandtl numbers. Finally in Fig. 5 the calculated Nusselt numbers are compared with the correlation recommended by the Engineering Sciences Data Unit (**18**):

$$St = \exp\{-3{\cdot}796 - 0{\cdot}205 \ln Re - 0{\cdot}505 \ln Pr - 0{\cdot}0225(\ln Pr)^2\} \qquad (6)$$

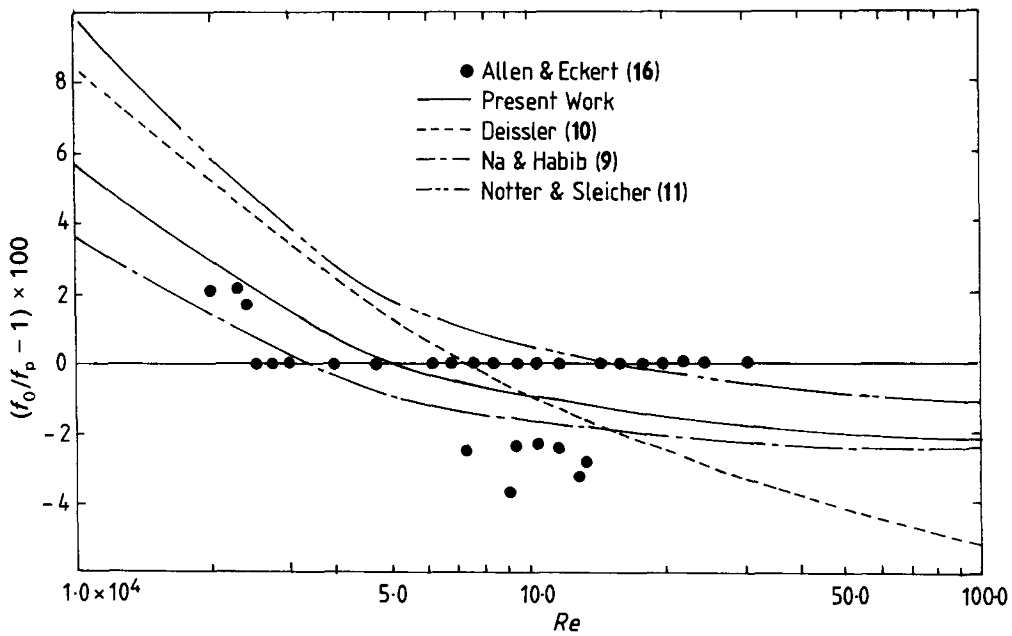

Fig. 3. Percentage deviation from Prandtl friction factor law for smooth pipes

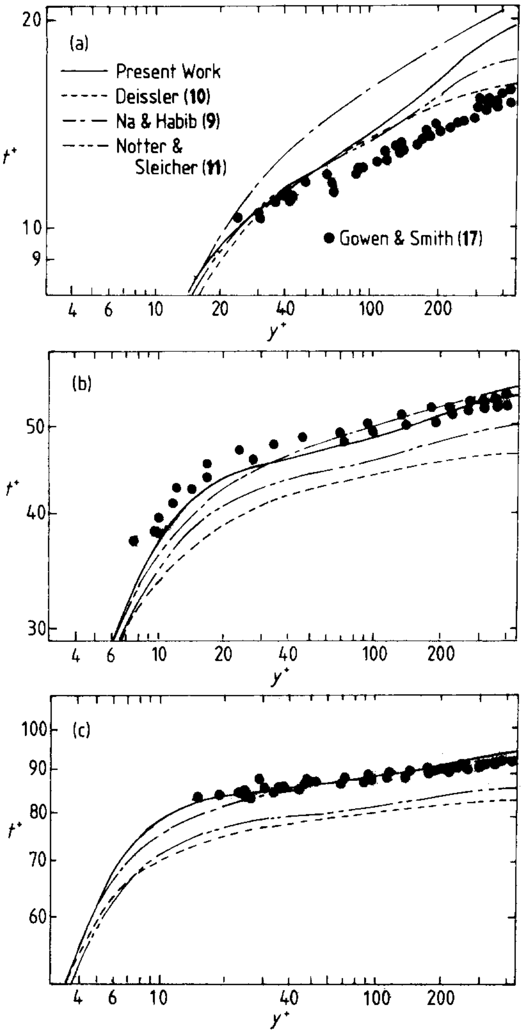

Fig. 4. Experimental and theoretical temperature distributions. (a) $Pr = 0{\cdot}7$; (b) $Pr = 5{\cdot}7$; (c) $Pr = 14{\cdot}3$

The following observations are made concerning Fig. 5:

(1) Equation (6) correlates the experimental data on which it was based with an overall r.m.s. error of 10·2 per cent; $\pm$ 10 per cent lines are included in Fig. 5.
(2) For Pr of 3 and 8 there was little experimental data beyond $Re = 10^5$ and the curves are shown dotted beyond that point.
(3) The data of Allen and Eckert (**16**) and Malina and Sparrow (**19**) merit special display because of the high precision of the experimental work. For Pr of 3 and 8 they show a trend similar to that of the theoretical curves; also at Pr of 48 and 75 (not shown).
(4) Nusselt numbers have been calculated using three alternative, published, eddy diffusivity formulations. These results are not included in Fig. 5 but they fit the experimental correlation rather less well than does the present formulation.

EXTENSION OF CALCULATIONS TO VARIABLE PROPERTIES

Introduction

Our initial objective was to extend the chosen constant property eddy diffusivity model logically and systematically to embrace each of the four physical property variations in turn. Unfortunately the quite arbitrary form of these eddy diffusivity models, including our own model, constituted a major barrier to our objective which grew in size on closer acquaintance. As these models have no fundamental basis it is difficult to devise logical arguments for their extension, and since they can be extended in an unlimited number of ways it is impossible to study them all.

Of the four properties, the variations of k and c_p, separately or in combination, present less of a problem than μ and ρ in that the momentum equation is not

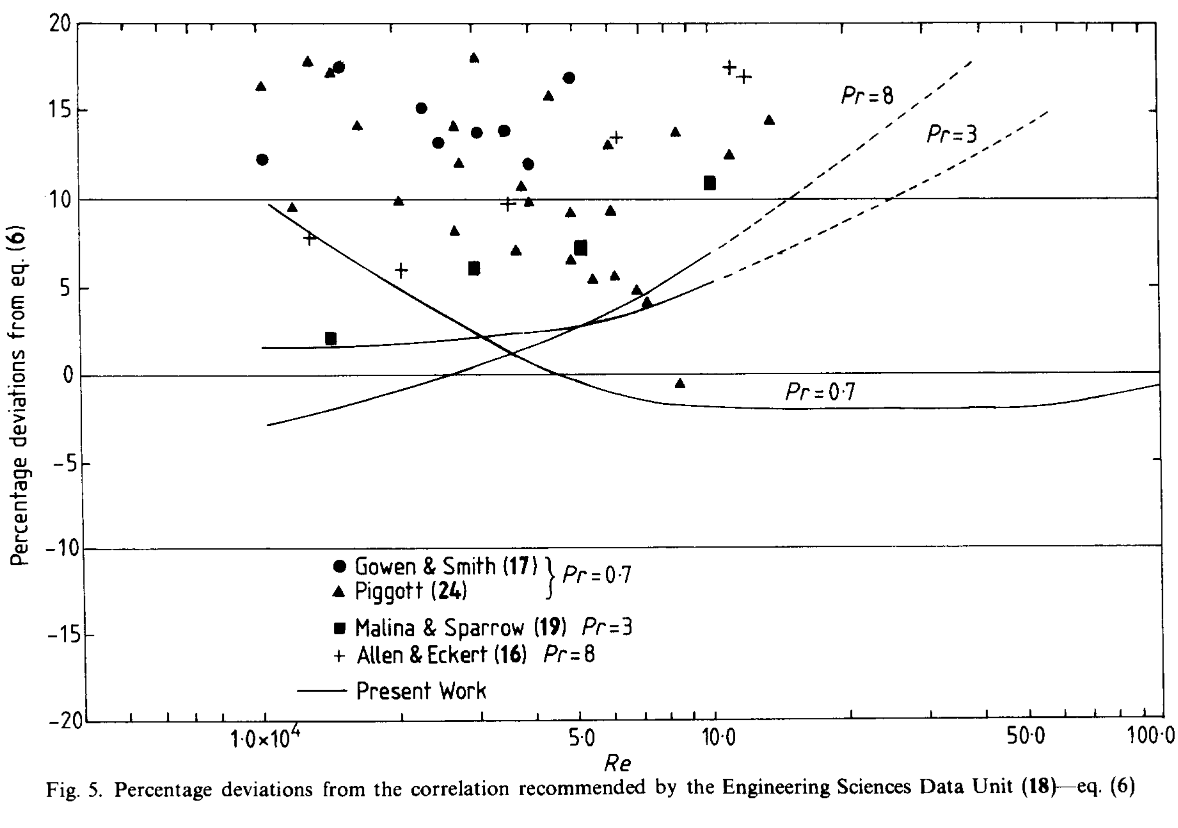

Fig. 5. Percentage deviations from the correlation recommended by the Engineering Sciences Data Unit (**18**)—eq. (6)

affected. It seems reasonable to use the local conductivity throughout and to have fair confidence in the results, bearing in mind that its main influence is in the viscous sublayer. It has also been decided, with rather less justification, to use the local specific heat.

In the case of ρ and μ variations the momentum and heat transfer equations are coupled and the problem is complicated. It has been found that extending the eddy diffusivity expressions by using the local ρ and μ wherever they occur produces results for Nu and f that do not agree with experimental data. Further arbitrary extensions of the eddy diffusivity expressions have therefore been introduced as will be explained below.

Thermal conductivity and specific heat variations

Extensive calculations have been made for Re from 10^4 to 10^6, Pr_b from 0·7 to 75, $\Delta c_p/c_{pb}$ and $\Delta k/k_b$ from −0·5 to 0·5. Only linear variations of c_p and k with temperature were considered as this constitutes a reasonable approximation to many practical cases for moderate property variations.

Typical results for the separate variations of c_p and k, for $Pr_b = 0{\cdot}7$ and two values of Re, are shown in Fig. 6. The results of these and similar calculations are generally as expected in that:

(1) The influence of thermal conductivity is appreciably greater than that of specific heat; in practice this is further emphasized by the greater temperature dependence of the thermal conductivity.
(2) The effect of the thermal conductivity variation increases with Pr and with decreasing Re as the viscous sub-layer accounts for a greater fraction of the total thermal resistance; the opposite trends apply to the specific heat variation.

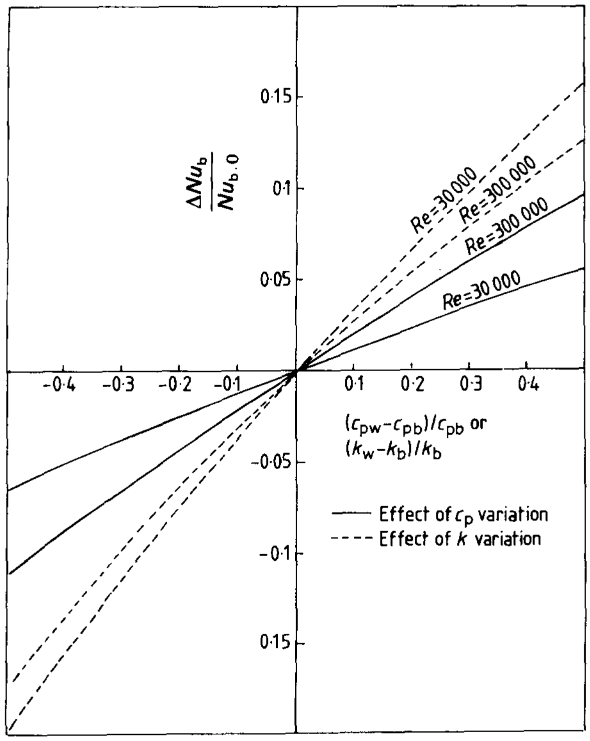

Fig. 6. Effect of separate variation of c_p and k (linear with temperature) on Nu for $Pr_b = 0{\cdot}7$ and $Re = 30\,000$ and $300\,000$

(3) As a consequence of (2) the results cannot be correlated, as is sometimes suggested, by extending constant property correlations by the factors $(c_{pw}/c_{pb})^a$ and $(k_w/k_b)^b$, where a and b are constants.

The results for combined (linear) variations of c_p and k can be quite well represented by a second-order polynomial, for example:

$$\frac{Nu_b}{Nu_{b,0}} = 1 + 0{\cdot}1836\frac{\Delta c_p}{c_{pb}} + 0{\cdot}3224\frac{\Delta k}{k_b} - 0{\cdot}0194\left(\frac{\Delta c_p}{c_{pb}}\right)^2 - 0{\cdot}0857\left(\frac{\Delta k}{k_b}\right)^2 + 0{\cdot}1048\left(\frac{\Delta c_p}{c_{pb}}\right)\left(\frac{\Delta k}{k_b}\right) \quad (7)$$

These particular coefficients apply for $Re = 10^5$, $Pr = 0{\cdot}7$. Note that when $(\Delta c_p/c_{pb}) = (\Delta k/k_b)$ the three second-order terms vanish and

$$\frac{Nu_b}{Nu_{b,0}} = 1 + 0{\cdot}5\frac{\Delta c_p}{c_{pb}}\left(\text{or } \frac{\Delta k}{k_b}\right) \quad (8)$$

This is part of a more general result for identical c_p and k variations which has already been reported **(25)** and which also leads to the conclusion that variable property Nusselt number data could not in general be correlated by the insertion into a constant property correlation of c_p and k values taken at some reference temperature.

Variable viscosity: comparison with data for liquids

It appears unlikely that the effects of a viscosity variation over the pipe radius can be adequately represented by using constant property eddy diffusivity expressions modified only by substitution of the local viscosity wherever it occurs. Interactions must be expected to occur involving the complete viscosity variation over the pipe radius. Resolution of this problem would require substantial experimental data on mean velocity and temperature distributions and turbulence quantities for liquid flows with heat transfer. At present the experimental data are limited to Nusselt numbers and friction factors. Fortunately they include the measurements of Nu and f, for $Pr = 8$, by Allen and Eckert **(16)** and of Nu, for $Pr = 3$, 48, and 75, by Malina and Sparrow **(19)**, which are unique by virtue of the thoroughness and precision of the experimental work. These experimental results were correlated by those authors by

$$\frac{Nu_b}{Nu_{b,0}} = \left(\frac{\mu_b}{\mu_w}\right)^{0{\cdot}05} \quad (9)$$

$$\frac{f}{f_0} = \left(\frac{\mu_w}{\mu_b}\right)^{0{\cdot}25} \quad (10)$$

There were insufficient data for an adequate examination of the possibility that the indices may depend upon Re and Pr.

Most of the experimental data on heat transfer and friction with variable properties has been obtained with heat flow from the pipe to the coolant because of the convenience of electrical heating. It is a good deal more difficult to obtain reliable data for cooled pipes and little exists, although it would be valuable. We have included in our comparisons the Nu data of Petukhov (**21**) for oil flow in cooled pipes and the f data of Rohonczy (**22**) for water flow in cooled pipes.

Calculations of Nu and f were tried using the constant property eddy diffusivity model but with the local viscosity wherever it occurs and, as anticipated above, the results were not in satisfactory agreement with the experimental data. A tentative assessment was then made of eddy diffusivity expressions containing a combination of μ_w, μ_b, and μ (the local value). The constant property μ was replaced by

$$\mu_w^\alpha \mu_b^\beta \mu^\gamma \qquad \text{where } \alpha + \beta + \gamma = 1$$

Different combinations of α, β, γ were considered and the results were compared with the Nu and f data of Allen and Eckert. The basic idea did not justify a length optimization exercise. The tentative conclusions were that γ should be about 0·9, i.e., the local viscosity predominates, α should be zero, indicating that the influence of the wall viscosity is confined to the wall, leaving $\beta = 0{\cdot}1$. This approach was not considered worth pursuing but it did lead to the decision to introduce μ_b into the eddy diffusivity expression in the form

$$\frac{\varepsilon_m}{\nu} = \left(\frac{\varepsilon_m}{\nu}\right)_L \left(\frac{\mu_b}{\mu}\right)^m \qquad (11)$$

where the value of m was to be optimized. Calculations of Nu and f over the full ranges of Re and Pr led to the choice of $m = 0{\cdot}5$.

For comparison with the experimental data for water ($Pr = 3$ and 8), $\mu \propto t^{-1{\cdot}1}$ was used to represent the viscosity data; for oil ($Pr = 48$ and 75) $\mu \propto t^{-3{\cdot}5}$. In fact the change of index has little effect on $Nu_b/Nu_{b,0}$ and f/f_0 for a fixed value of μ_w/μ_b. Some of the calculations are compared with experimental data in Fig. 7. The agreement on f is good. The agreement on $Nu_b/Nu_{b,0}$ for liquids in heated pipes ($\mu_w < \mu_b$) is not good; the calculations correspond to $(\mu_b/\mu_w)^{0{\cdot}16}$, the experimental data to $(\mu_b/\mu_w)^{0{\cdot}05}$. For cooled pipes ($\mu_w > \mu_b$) the theoretical curve for $Nu_b/Nu_{b,0}$ passes through a minimum and then unexpectedly rises above unity. The cause of this is that the increasing f becomes the dominant factor in the eddy diffusivity expression. Although the experimental data do not support this trend, they are too scattered to refute it clearly. Inspection of the results of calculations for the full range of Re and Pr, and their comparison with experimental data for liquids and for gases (embracing also, density variations to be discussed below) indicated that $m = 0{\cdot}5$ in eq. (11) was about the optimum choice, but that the form of eq. (11) leaves something to be desired. It is possible that the discrepancies on Nu could have been reduced, without disturbing the agreement for f, by manipulating the formulation for Pr_t and its dependence on the property variations. Information on Pr_t and its dependence on Pr is contradictory, and further arbitrary modifications have not been pursued. Local properties have been used throughout in the evaluation of Pr_t.

It has often been suggested that the correlation of experimental data involving property variations can be achieved by using constant property correlations with the properties taken at some reference temperature, t_r, between t_b and t_w. Thus if the constant property friction factor data are represented by

$$f_0 = CRe^{-s}$$

the non-isothermal friction factor data for liquids would be correlated by

$$f = C\left(\frac{\rho u d}{\mu_r}\right)^{-s} = CRe_b^{-s}\left(\frac{\mu_b}{\mu_r}\right)^s \qquad (12)$$

Thus according to this proposal the maximum conceivable effect of the property variations is given by setting $\mu_r = \mu_w$. The isothermal friction data of Allen and Eckert (**16**) agree very closely with the Prandtl formula (5) which, for $20000 < Re < 110000$, is equivalent to $f_0 = CRe^{-0{\cdot}225}$. The effect of the viscosity variation should therefore be less than $(\mu_b/\mu_w)^{0{\cdot}225}$ according to the reference property approach. In fact Allen and Eckert found a rather greater effect, $(\mu_b/\mu_w)^{0{\cdot}25}$; our calculations also have produced friction factor changes arising from viscosity variations which are greater than could possibly be represented by the reference property approach. On reflection it is not surprising that a low value of μ_w at the wall coupled with a higher viscosity in the turbulent core can produce a friction factor lower than would result from any uniform viscosity between μ_b and μ_w.

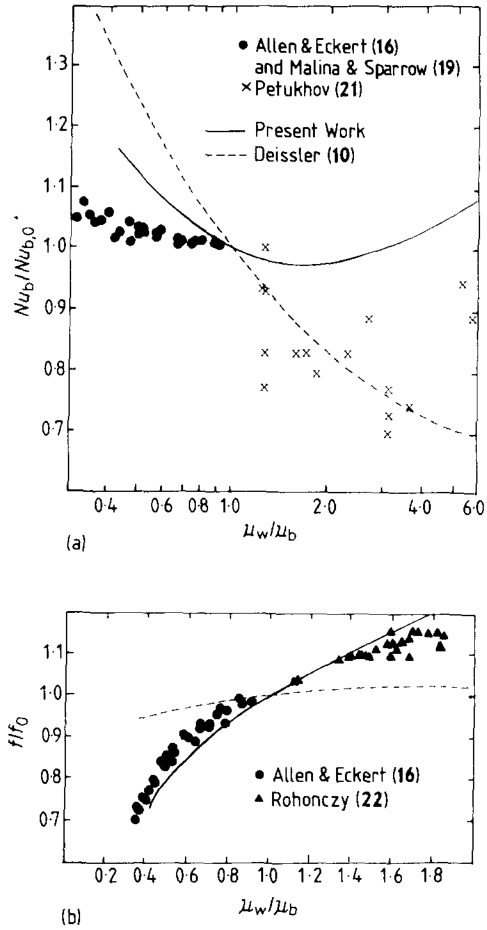

Fig. 7. Comparison between theory and experiment for the effect of viscosity variation on: (a) Nusselt Number, (b) Friction Factor

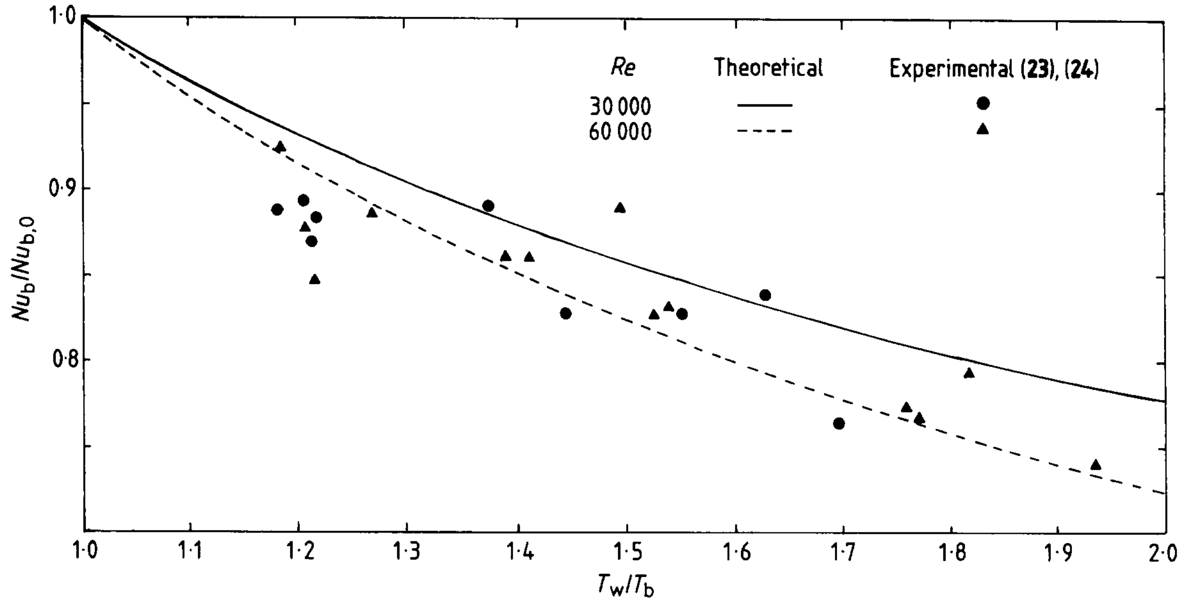

Fig. 8a. Variation of $Nu_b/Nu_{b,0}$ with T_w/T_b for air

Density variation: comparison with data for gases

One of our principal objectives was to isolate the effects of the variations of k, c_p, μ, and ρ. So far the first two have been studied theoretically, and μ by reference to experimental data for liquids. The density effect must be obtained from experimental data for gases, although these contain also the effects of significant k, μ, and sometimes c_p, variations. It might be supposed that the effects of k, μ, and c_p could be removed using the eddy diffusivity expressions established so far, in order to isolate the density effect. However, what remains after such a procedure is not the influence of the density variation alone but also contains the interactions between the variations of ρ, k, c_p, and μ, which calculations suggest may be significant. We have therefore resorted to a further arbitrary extension of the eddy diffusivity model:

$$\frac{\varepsilon_m}{\nu} = \left(\frac{\varepsilon_m}{\nu}\right)_L \left(\frac{\mu_b}{\mu}\right)^{0.5} \left(\frac{\rho}{\rho_b}\right)^n \tag{13}$$

by analogy with the procedure adopted for viscosity, with the index n to be optimized by comparison of predicted and experimental results for gases. We have concentrated particularly on the experimental investigations of Barnes and Jackson (**23**) and Piggott (**24**). Both are for moderate values of T_w/T_b and cover a number of different gases in the same apparatus; Piggott has measured both heat transfer and friction.

For air, nitrogen, helium, and argon, which were used in the above investigations, the variations of k_w/k_b, c_{pw}/c_{pb}, μ_w/μ_b and, of course, ρ_w/ρ_b with T_w/T_b are so very nearly independent of the gas that virtually identical Nu/Nu_0 and f/f_0 would be expected. That this is not always reflected in the experimental results is an indication of the difficulty of precise experimental work at relatively high temperatures. For carbon dioxide the variations of c_p and k are substantially greater.

Figures 8(a) and (b) show the variation of Nu with T_w/T_b for air and carbon dioxide respectively, the theoretical curves being obtained with $n = 1{\cdot}0$ in eq. (13). The theoretical and experimental results are in broad agreement. However, while the theoretical curves show an effect which increases with Re, no such effect is apparent in the experimental results for air; and the experimental points for carbon dioxide suggest the opposite trend.

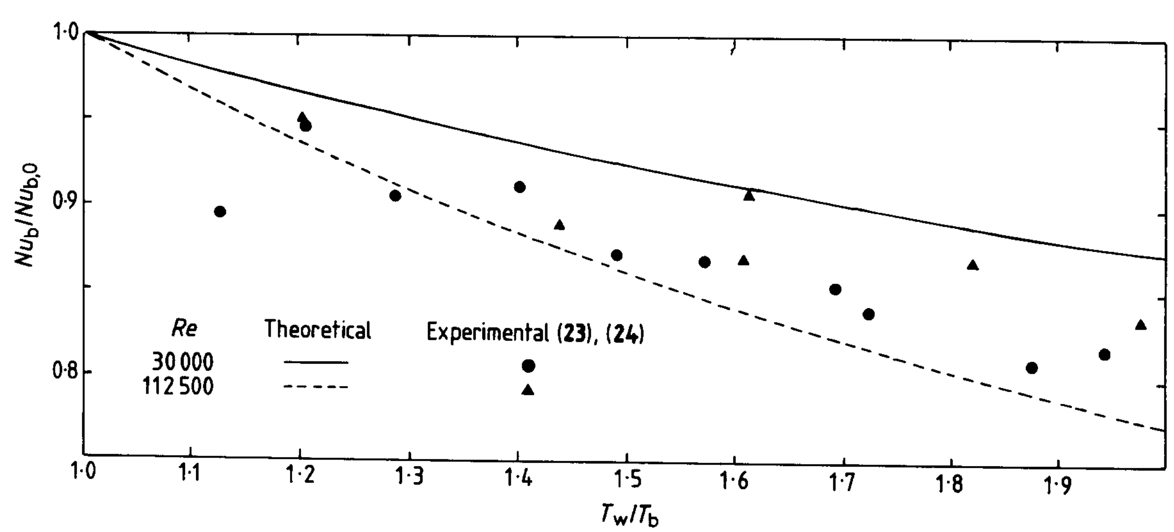

Fig. 8b. Variation of $Nu_b/Nu_{b,0}$ with T_w/T_b for carbon dioxide

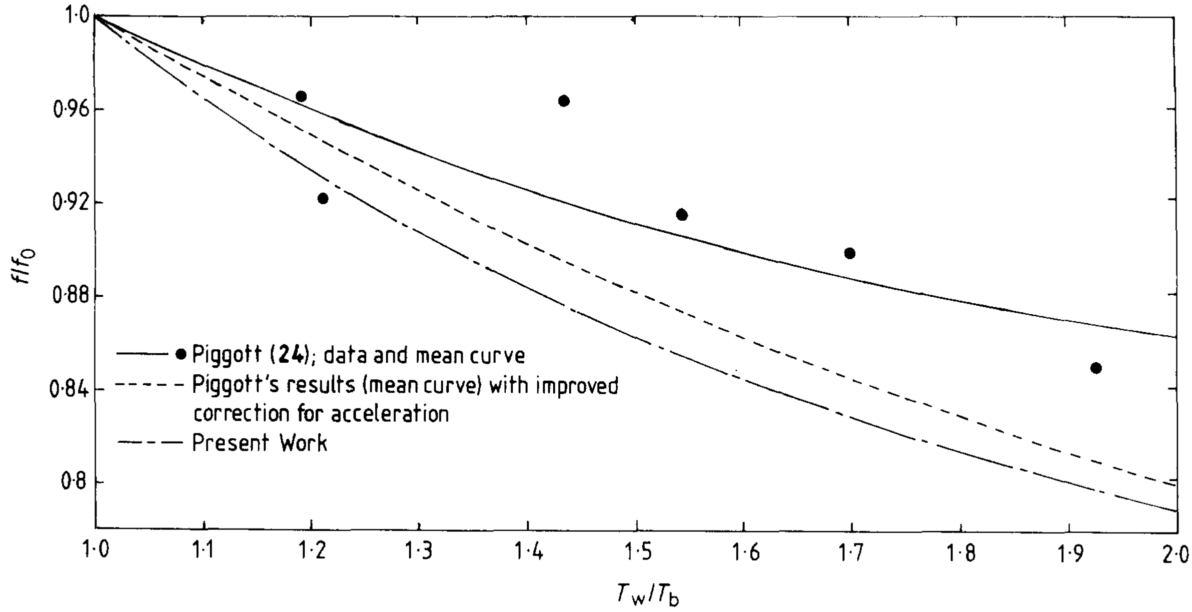

Fig. 9. Variation of f/f_0 with T_w/T_b for air; $Re = 28000$

Friction factors are calculated from experimental pressure drop measurements after subtraction of the acceleration component of the pressure drop, which becomes comparable with the friction pressure drop for $T_w/T_b = 2{\cdot}0$. It is necessary to calculate the increase in momentum flux over the test section and for this purpose Piggott assumed slug flow with $\mu = \mu_b$ and $\rho = \rho_b$. We have recalculated the acceleration pressure drop with proper velocity and density distributions and find that it is substantially greater than the slug flow approximation. Figure 9 shows:

(1) Piggott's experimental data for air, $Re = 28000$, based on the slug flow approximation.
(2) A mean curve through Piggott's data to which has been applied the more accurate correction for acceleration.
(3) A theoretical curve for f/f_0 with $n = 1{\cdot}0$ in eq. (13).

It is seen that the prediction is in fair agreement with the corrected experimental data. The choice of $n = 1{\cdot}0$ is confirmed by a wider range of heat transfer and friction calculations **(20)**.

CONCLUDING REMARKS

Eddy diffusivity models involving expressions of plausible form together with adjustable constants have limited value for predictive purposes. This is hardly an original observation but one of which we have become more keenly aware during attempts to extend such expressions to the variable property problem, resulting in arbitrary and simplistic approximations.

However, the multiplicity of experimental correlations are at least as unsatisfactory, particularly for gases, frequently involve implicit and unsubstantiated assumptions, and depend upon inadequate and often contradictory experimental data. No great reliance can be placed upon them for design purposes.

Little real progress seems possible without further experiments with gases of precision comparable with that achieved by Allen and Eckert for liquids, and preferably with more detailed measurements such as mean velocity and temperature distributions and turbulence parameters to provide the basis for a more fundamental approach.

In the meantime theoretical results derived from the present eddy diffusivity model should provide guidance at least as useful as that available from published correlations.

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